How far away is the earthquake?

Most everyone will recall that the TRAVEL TIME, T of a wave (or a car, train, etc) is:

$$T = \frac{X}{V} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

So, if we knew the travel-time and speed of a wave, we could calculate the distance to the earthquake that generated that wave.

But, for earthquake waves, we can't observe T directly. All we know is the ARRIVAL TIME at our station, usually measured relative to Greenwich Mean Time. The problem is, we don't know WHEN the earthquake occurred. This unknown is called the ORIGIN TIME of the earthquake. The relationship between TRAVEL TIME, ARRIVAL TIME, and ORIGIN TIME is:

$$\text{ARRIVAL TIME} = \text{ORIGIN TIME} + \text{TRAVEL TIME}$$

Fortunately, at any station, we can measure the ARRIVAL TIME for two seismic waves, the P-wave and the S-wave, that both leave the hypocenter at the same origin time and travel the same distance. We use the symbols A_p and A_s for the P-wave and S-wave ARRIVAL TIMES, respectively. We use O for the origin time. Finally, we use T_p and T_s for the P-wave and S-wave TRAVEL TIMES, respectively. Then, we write.

$$A_p = O + T_p$$

$$A_s = O + T_s$$

If we subtract A_p from A_s, we can eliminate the unknown ORIGIN TIME:

$$A_s - A_p = (O + T_s) - (O + T_p) = T_s - T_p$$

This shows that even though we still do not know when the earthquake occurred, we can still use the observed difference in ARRIVAL TIMES of the P-wave and S-wave at a station to find out how far away the earthquake is from our station. To see this, we use V_p for the P-wave speed and V_s for the S-wave speed and write:

$$T_p = \frac{X}{V_p} \text{ and } T_s = \frac{X}{V_s}$$

and we can see that:

$$A_s - A_p = A_{s-p} = \frac{X}{V_s} - \frac{X}{V_p}$$

To simplify this, we find a common demoninator on the right-hand side:

$$A_{s-p} = \frac{XV_p}{V_pV_s} - \frac{XV_s}{V_pV_s}$$

and finally:

$$X = \frac{V_pV_s}{V_p - V_s} A_{s-p}$$

This equation can be used to find the distance from an earthquake to a seismic station assuming we can measure the arrival times of the P-wave and S-wave and we know the speed of each of those waves. As we will learn later, the wave speeds are not always constant or well-known. This can be a source of error. For nearby earthquakes, assuming a P-wave speed of 6.5km/sec and an S-wave speed of 3.8km/sec is a reasonable assumption.